Publications Committee

University of Texas Bulletin

No. 1719: April 1, 1917

The Texas Mathematics Teachers' Bulletin

(Vol 2, No. 3, April 1, 1917)

Published by the University six times a month and entered as
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AUSTIN, TEXAS

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The University publishes bulletins six times a month, so numbered that the first two digits of the numbers show the year of issue, the last two the position in the yearly series. For example, No. 1701 is the first bulletin of the year 1917. These comprise the official publications of the University, publications on humanistic and scientific subjects, bulletins prepared by the Department of Extension and by the Bureau of Municipal Research, and other bulletins of general educational interest. With the exception of special numbers, any bulletin will be sent to a citizen of Texas free on request. All communications about University publications should be addressed to the Editor of University Publications, University of Texas, Austin.

A. C. BALDWIN & SONS: AUSTIN

University of Texas Bulletin

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The Texas Mathematics Teachers' Bulletin

(Vol 2, No. 3, April 1, 1917)

Edited by

J. W. CALHOUN
Adjunct Professor of Pure Mathematics

and

C. D. RICE
Associate Professor of Applied Mathematics

This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

MATHEMATICS TEACHERS' WEEK

To the Teachers of Mathematics of the State of Texas:

There will be a conference on the teaching of Secondary Mathematics held at the University of Texas June 13 to 16. It will be held in the Engineering Building, Rooms 207 and 305.

In Room 207 will be held daily lectures on the various branches of mathematics taught in the secondary schools. There will be a series of lectures on Plane Geometry by H. Y. Benedict, Professor of Applied Mathematics, a series on Algebra by J. W. Calhoun, Adjunct Professor of Pure Mathematics, a set of special lectures on computation, the graph, and some other special topics by other members of the faculties of Pure and Applied Mathematics.

Room 305 of the Engineering Building will be set aside for library purposes during the week. Professor A. A. Bennett will collect and organize there a mathematical library. Printed bibliographies, references and helps will be furnished the readers. The library will be in charge during the week of Professor C. D. Rice. He will make it his business to see to it that people who want information as to books on teaching or on any branch of mathematics may be helped to find what they want with the least loss of time.

There will be other features but we deem these enough to set down here. We hope to have a large attendance and a profitable time for all who can attend.

Last year saw the first effort made to get the teachers of the state to take counsel as to the state of things, to discuss our problems, and render mutual help to one another. Though the attendance was not very large those who came declared the meeting a beneficial one and entered into an agrement to return this year, each to bring four others with him. It is easy to see that if they live up to this and the same missionary spirit should animate those who attend for a few consecutive years that we should soon have as many people at the Conference as it cost dollars to shoe the horse we used to cipher about where the first nail cost one cent, the second three cents, the third nine cents, etc., where we got an answer bigger than the national debt of all the European countries put together will be after the close of the war.

We hope these lines will meet the eye of each one who came last year and that it may remind him of his promise if, perchance, he may have forgotten. Let no one who came last year fear that he will hear the same teachers he heard then. There will be a complete change of program. The lectures may be worse than last year—we hope not—they will be less numerous, and they will be different.

If any teacher of mathematics in Texas thinks he is so efficient and well equipped as not to need this conference we beg him to come and help out the rest of us who are not so well off; if any thinks he is so poorly off that he will be at a disadvantage in comparison with the rest, we beg him to come and assure him that if he is at all prudent his deficiency will never be discovered and it is barely possible he may discover that the help he will get will be in direct proportion to his needs. In short, we give a pressing invitation to all teachers of mathematics to come, there is no fee, no tuition, no red tape, no collection, nothing to sell, nothing to subscribe to, no book agents allowed on the premises during the days of June 13 to 16.

Come down and let us just revel for a few days in the associations of our fellows and the getting of new views and some new inspiration in the subject close to the hearts of all of us.

RATIO

C. D. RICE

The simplest use of the letter to represent a number in arithmetical processes is found, probably, in the usual treatment of ratio and proportion. In fact we find most authors of arithmetic use the letter for the unknown even where no knowledge of the equation has been previously given. A little training as outlined in previous numbers of this bulletin on the subject of Literal Arithmetic will give ample preparation for the study of ratio and proportion.

The quotient of one quantity by another like in kind is called the *ratio* of the first to the second.

Thus the ratio of \$7 to \$3 is 7/3
and the ratio of \$*a* to \$b is a/b

The ratio of two quantities may be defined, also, as the quantitative comparison of the first to the second where the second is taken as the *standard* or *unit* by which the first is measured.

Since the quotient of one quantity by another like in kind is an abstract number, every ratio has a *definite numerical* value.

Thus the ratio of 5 ft. to 2 ft. is $\frac{5 \text{ ft.}}{2 \text{ ft.}} = \frac{5}{2}$

and the ratio of 5 yds. to 2 yds. is $\frac{5 \text{ yds.}}{2 \text{ yds.}} = \frac{5}{2}$

These results show that any ratio derived from a comparison of two like quantities is an *abstract number* or as stated above every ratio has a numerical value.

The ratio of *a* to *b* is usually written

$$\frac{a}{b}$$

but the forms

$$a:b \text{ and } a \div b$$

are often used.

Example. Find the quantity that has the ratio 5:6 to $18.

From the definition of a ratio we know the required quantity is a certain number of dollars.

Let x represent the required number of dollars. Then

$$\frac{\$x}{\$18}=\frac{5}{6} \text{ or } \frac{x}{18}=\frac{5}{6}$$

Multiplying both sides of this last equality by 18 we have

$$x=15$$

Hence $15 is the quantity required.

Example. What number x will make the ratio x+6:9 equal to 3:4.

By the conditions we have

$$\frac{x+6}{9}=\frac{3}{4}$$

Multiplying both sides of this equality by 36 we have

$$4x+24=27$$
$$\text{or } 4x=3$$
$$\therefore x=\tfrac{3}{4}$$

With a proper selection of problems, the pupil may become familiar with methods of solving simple equations without having the negative number to appear, and in this way become accustomed to the use of letters as numbers before taking up the negative number.

In beginning the study of ratio the teacher may give examples to the class like the following:

(1) What is the value of x in

(a) $\frac{x}{a}=5$ when $a=2$, $a=1$, $a=5$

(b) $\frac{a}{x}=5$ when $a=2$, $a=1$, $a=5$

(c) $\frac{a+11}{x}=4$ when $a=2$, $a=3$, $a=5$

(d) $\frac{x}{a+5}=7$ when $a=3$, $a=5$, etc.

(2) Find the quantity that has the ratio

(a) of 3:4 to 16 ft.

(b) of 2:5 to $12\frac{1}{2}$ acres, etc.

(3) What is the value of x in each of the ratios

(a) $\frac{x}{5}=7$

(b) $\frac{x+3}{4}=5$

(c) $\frac{2}{x+7}=\frac{3}{14}$

(d) $\frac{21+x}{11+x}=\frac{5}{3}$ etc.

One of the simplest and most useful applications of ratio is that of *Specific Gravity.*

The specific gravity of any quantity is defined as the ratio of the weight of a given volume of the quantity to the weight of a like volume of another quantity taken as a standard. For heavier substances distilled water is usually taken as the standard.

Example. If the weight of a cubic foot of distilled water is 62.5 pounds and the weight of a cubic foot of mercury is 849.75 pounds, what is the specific gravity of mercury?

According to the definition of specific gravity we have

$$x=\frac{849.75}{62.5}=13.596$$

as the specific gravity of mercury.

Example. What is the weight of a cubic foot of silver if its specific gravity is 10.5?

If we let x represent the weight in pounds we have

$$\frac{x}{62.5}=10.5$$

$$\begin{aligned}x&=62.5\times10.5\\&=656.25\end{aligned}$$

THREE ESSENTIALS FOR THE MATHEMATICS STUDENT WHO IS LOOKING FORWARD TO ENGINEERING

H. J. ETTLINGER

In the folowing it is aimed to point out a few desirable points of view which the writer has remarked to be conspicuous by their absence among the freshmen in his engineering classes. They represent an attitude which it would be well to cultivate, not alone among those who are looking forward to the engineering profession, but as a discipline for all students.

Engineering is an ancient and honorable profession. The first mathematicians may have been alchemists and philosophers in their old age, but it is not at all unlikely that they were dependent for their livelihood in those bread and butter days on the applications they could make to the solution of practical problems in the peaceful or the military arts. Archimedes and Pythagoras were pre-emiently occupied with problems which today furnish us with the principles which underlie our elementary phases of algebra, geometry, and trigonometry. Thales of Miletus, so Plutarch tells us, measured the distances of ships at sea by a system of similar triangles.

Today, algebra and geometry have become so much a matter of the study of abstract principles and the manipulation of symbols that the source of these problems in the earlier attempts to devise convenient and practical methods of computing is almost completely lost. But from its dim beginnings by the Euphrates and the Nile, mathematics has been on the one hand a means by which man has constantly increased his understanding of his environments and his power of manipulating it, on the other hand a body of pure ideas. This *two-fold* significance the pupil should be made to realize, the first should not be lost sight of for the benefit of the second, and the practical background should be ever kept in mind.

Along with this practical background there should be cultivated the regular habit of checking the correctness of a result.

While in the business world an error may mean the loss of dollars, in problems of engineering it may mean the loss of lives as well. The *habit of checking a result,* whether it be the solution of an equation, or the result of numerical computation, should be instilled in the minds of the aspiring engineer. In the case of an equation the substitution of the answer into the original equation to be solved should be part of the problem. In his early days of mathematical studies the student's problems are arranged frequently so that they may come out "nice and even." And if they come out this way the student usually assumes his answer to be correct, despite the fact that frequently the result arrived at may either be incorrect due to an error in reasoning or in computation, or else the root does not belong to the original equation.

Where a direct check in the nature of a verification by substitution may not be had the habit should be cultivated of checking up the result by estimating the probable value of the result from the original data. In logarithmic computations also, this method is desirable and useful, and will invariably remove the possibility of an error in the position of the decimal point.

A third habit that should be cultivated in the minds of every student is a neat and orderly manner of setting down the work. *Systematic arrangement* makes for clearness in analysis as well as lessening opportunities for incorrect computation.

These then, are the three essentials in moulding in particular the plastic mind of the future engineer and that of the general student as well:

(1) The practical background.
(2) The check.
(3) The systematic arrangement.

CORRELATION TABLES PUBLISHED IN 1916 BY PROF. TRUMAN LEE KELLEY, ADJUNCT PROFESSOR OF THE PHILOSOPHY OF EDUCATION, UNIVERSITY OF TEXAS

EDWARD L. DODD

Many teachers of mathematics are unaware of the vast progress made in mathematics in the direction of laying the foundations for important investigations in other fields of research. They are unaware that specialists in seemingly remote subjects are trained mathematicians. They are unaware that "Science" no longer means "Natural Sciences" but includes Social Science, Psychology, and Education, where intensive research by scientific methods is being conducted,—research often qualitative in its first stages, but becoming more and more quantitative, requiring more and more mathematics.

The Texas Mathematics Teachers' Bulletin takes pleasure in presenting to its readers the introductory portion of a bulletin written by Prof. Kelley of the Department of Education at the University of Texas. The title of this bulletin is: "Tables: To Facilitate the Calculation of Partial Coefficients of Correlation and Regression Equations." This bulletin is No. 27 for the year 1916. To facilitate the use of the tables, the bulletin outlines the method of calculation and gives illustrative examples fully worked out. The purpose of the tables is set forth in the introductory pages which are here reproduced.

Introduction

In these tables Dr. Kelley has done much to make practicable a wider use of certain methods of statistical analysis of related facts developed by Edgeworth, Pearson and Yule.

The mere title of these tables and an inspection to make sure that they are derived with precision will secure a welcome for them from experts in modern statistical methods, who understand the importance of partial correlation and have been prevented from using the method by the elaborateness of the computations required. The tables will reduce this by about eighty per cent.

The number of students of the biological and social sciences who do understand the importance of partial correlation coefficients or regression equations is, however, small because of the recency of the development of the mathematical technique as well as its intricacy.

Yet we all need command of this delicate instrument for analyzing resemblances or correspondences, if we are to utilize quantitative data fully. Indeed it is in some of the most practical fields of investigation that it is most needed. The economist who seeks the causes of the high cost of living, the psychologist who analyzes the merit of advertisements, the educational expert who gives vocational guidance—these are samples of the many workers in the social sciences who should master the theory if they can, and should at least learn how to use the formulae and interpret their results.

Twenty years ago the ordinary coefficient of correlation for one series of paired values was a recondite mathematical technique just beginning to be used by a few biometricians. Now it is a stock means of measuring resemblance or correspondence employed when appropriate by all competent investigators. The same career might be prophesied for partial coefficients of correlation, but for the complexity and tediousness of the computations involved. It is the service of Dr. Kelley's tables to reduce these.

There is, of course, some danger that mathematical devotees may use this statistical analysis in cases where experimental analysis would be sounder, and that careless thinkers may use it where it is inappropriate. On the whole, however, it seems certain that the prophecy of the influence of factors *per se* by means of regression equations will stimulate rather than repress efforts to isolate the factors by experiment; and that careless thinkers will do less harm with partial correlation coefficients than without them.

EDWARD L. THORNDIKE,
Teachers College, Columbia University.

Author's Preface

The regression equation method has been so laborious, as well as involving such accuracy in and knowledge of statistical method that it has not been used in many studies in which it alone could evaluate the data in such a manner as to answer the questions involved. It is hoped that the tables here presented have so materially decreased the labor of calculation that the method will be used extensively. If this has been accomplished a second edition will be demanded and if such is called for two important improvements may be expected, first, that the tables be carried at least two decimal places further and, second, that entries for at least one of the variables be for every .001 instead of as at present for every .01. The shortcomings mentioned are well recognized. The author would be glad to hear of any others discovered by users. T. L. K.

THE FUNCTION OF PARTIAL COEFFICIENTS OF CORRELATION AND REGRESSION EQUATIONS

(1) Partial coefficients of correlation.

If several measures are correlated with a common measure and with each other, then the coefficient of correlation between any one of these several measures and the given measure represents the correlation that exists due to itself and due also to the indirect effect of the other measures. To cite a familiar example: if alcoholism is correlated with degeneracy to the extent r_{ad} and if bad heredity is correlated with degeneracy to the extent r_{hd} and if alcoholism is correlated with bad heredity to the extent r_{ah}, then the correlation between alcoholism and degeneracy, r_{ad} does not measure the effect of alcoholism alone upon degeneracy, but it is a function of the effect of alcoholism plus the indirect effect of heredity upon degeneracy and this indirect effect may be the one of major importance.

To determine the relation between alcoholism and degeneracy independent of heredity it is necessary to find the partial coefficient of correlation, $r_{ad.h}$ which may be variously read "the correation between alcoholism and degeneracy, heredity eliminated,'' or, ''the corelation between alcoholism and degeneracy in the field of heredity,'' or, ''the correlation between alcoholism and degeneracy, heredity constant.'' Whenever two factors, such as a and h, are correlated with each other and with a third, such as d, the partial correlations, such as $r_{ad.h}$ and $r_{hd.a}$ are demanded for purposes of analysis and understanding of causal relationships. No coefficients of correlation, not even partial coefficients, necessarily measure causal relationships, but a partial coefficient of correlation does assist in determining such relationships by giving a measure of existing relationships when the relation of other factors has been eliminated.

A partial coefficient of higher order, such as $r_{ad.he}$ has a comparable significance and, if ''e'' stands for environment, means the correlation between alcoholism and degeneracy having eliminated the effect of heredity and environment.

Partial coefficients of correlation are, then, in and of themselves, of intrinsic merit, in addition to the function they perform in the regression equation.

(2) Regression Equations:

If three sets of measures are inter-correlated, such as measures of accomplishment in English, of intellectual ability and of conscientiousness, and it is desired to estimate with as high a degree of accuracy as possible one of the measures by means of the other two, e. g. to estimate accomplishment in English from measures of intellectual ability and of conscientiousness, it is necessary to combine the latter two measures into a single one in such a way that they give the closest possible estimate of accomplishment in English, or, in other words, that they correlate to as high a degree as possible with accomplishment in English. This best combination must be such that it weights intellectual ability according to its correlation with English when conscientiousness is eliminated, i. e. accordng to $r_{ei.c}$, and similarly weights conscientiousness according to $r_{ec.i}$, provided each is measured in terms of its independence of the other two measures, or, in other words, in terms of its dependence upon itself alone. Expressed as an equation: (Standard deviations, for simplicity, assumed equal to 1.)

$$\text{Constant} \times e = r_{ei.c} \frac{i}{\text{dependence of i upon itself}} +$$

$$r_{ec.i} \frac{c}{\text{dependence of c upon itself}}, \text{ or this may be stated:}$$

$$\text{Constant} \times e = r_{ei.c} \frac{i}{\text{independence of i from e and c}} +$$

$$r_{ec.i} \frac{c}{\text{independence of c from e and i.}}.$$

These denominators—coefficients of independence—are just as important as measures of dependence—coefficients of correlation; they equal in fact $\sqrt{1\text{-}r^2}$, and are zero for perfect dependence and 1 for perfect independence.

It may be shown that if any other weighting is given to these measures than that indicated,* the combined measure (the entire right hand member of the equation) would, of necessity, correlate less highly with e. This is to say that if the weights of i and c are not weights according to their importances independent of the other variables, then one or the other has been overweighted. Expressing the last equation in more definite terms and giving the correct value to the constant which multiplies e, gives:

$$\frac{1}{\sqrt{1-r^2_{ei}} \quad \sqrt{1-r^2_{ei.c}}} e = \frac{r_{ei.c}}{\sqrt{1-r^2_{ei}} \quad \sqrt{1-r^2_{ic.e}}} i +$$

$$\frac{r_{ec.i}}{\sqrt{1-r^2_{ec}} \quad \sqrt{1-r^2_{ic.e}}} c, \text{ or, as usually expressed:}$$

$$e = (r_{ei.c} \frac{\sigma_{e.ic}}{\sigma_{i.ec}}) i + (r_{ec.i} \frac{\sigma_{e.ic}}{\sigma_{c.ei}}) c.$$

*Yule, Introduction to the Theory of Statistics, gives a proof not involving calculus.

The magnitudes in the parentheses are designated by the symbols $b_{ei.c}$ and $b_{ec.i}$ respectively, and are called regression coefficients.

The peculiar value of the regression equation is twofold:

(1) It gives the mathematical means of combining any number of measures into a single measure in such a way that the highest possible correlation with a dependent measure is obtained. If a regression equation combining several measures with reference to another measure has once been calculated, such an equation may be used in the future to estimate the dependent measure, provided the others are already known; for example, in a prognosis problem where several known factors contribute to a future result the regression equation provides the means of arriving at the closest estimate. This fact is of particular value when there are quite a number of contributory factors, for the problem quickly becomes too complex for the mind to cope with without mathematical aid in summing influences.

(2) Further, the regression equation makes possible an analysis of the relative importance of the various contributory factors which bear upon a final result.* This is readily seen to be so from the fact that the regression coefficients give weightings to the various factors that are proportionate to their significance independent of the other factors.

It may thus be pointed out that the regression equation is of peculiar service in combining several factors which contribute to the determination of the value of a further measure in all statistical work in biology, sociology, economics, psychology and education, where mutual implication exists between measures rather than invariable relationships, as in physics.

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*For this particular purpose standard deviations must be taken as equal in calculating regression coefficients.

THE QUADRATIC EQUATION

GOLDIE HORTON

It is deplorable that practically no college freshman is able to solve a quadratic equation and that his instructor is obliged to spend much time on this topic. This inability on the part of the average freshman is due partly to the fact that in the two or more years that have elapsed since he studied algebra he has forgotten the little that he once knew, and partly to the faulty presentation of the subject in most text-books. It was thought that the following remarks on the quadratic equation and its solution might stimulate a more systematic, a more careful presentation of the topic on the part of the readers of The Bulletin.

I. The study of the quadratic equation should be prefaced somewhat as follows: First, remind the student of what is meant by a polynomial; point out that it may have any number of terms, that it may be in any number of variables, and that it may be of any degree in any one variable, or in any set of variables. Cite many examples of polynomials, describing each example completely as to the number of terms, the number of variables, and degree.

Second, define an *equality* as the mere statement in symbols of the equality of two expressions. Point out that equalities fall naturally into two big classes: one the identical equality or the *identity*, examples of which are afforded by all the problems in simplifications, by all the cases of factoring; the other the conditional equality or *equation*. Give many illustrations. Mention that a polynomial in one variable written equal to zero is a very special type of an equation, that by definition its degree is the degree of the polynomial.

Third, define *root of an equation*, namely, a value of the variable which is such that, when substituted for the variable in the equation, the right and left hand members of the equation are identical. It is well to point out in this connection that an equation in one variable has *roots* while an equation in two or more variables has *solutions*. For example, $2x-6=0$

has 3 for a root, while $x+7y=15$ has $x=o$, $y=2\ 1/7$; $x=1$, $y=2$ etc. as solutions. Permit the student to shorten the definition of a root by saying a *root* of an equation is a value of the variable which *satisfies* the equation, the term *satisfy* having an evident interpretation. The student should be taught to say that 3 is a root of $2x-6=0$, that $(x-a)\ (x-b)=0$ has roots a and b. Emphasize that "to solve an equation" means "to find its roots," and insist upon the roots obtained being tested, that is, actually substitute for the variable in the equation the values claimed to be the roots, and, if the equation is satisfied accept them, and if it is not satisfied there is an error.

Fourth, emphasize the fact that if it is known that the product of several numbers is zero, then at least one of the numbers is zero. For example, if it is known that $M\ N=0$, then either $M=0$, or $N=0$, or both $M=0$ and $N=0$. This is an important fact and must be borne in mind.

II. Begin the study of the quadratic equation in one variable by the definiton: *a quadratic equation in one variable, x say, is an equation of the second degree in x,* that is, an equation in which x occurs to the second but to no higher power. Point out that according to the definition a quadratic equation must contain a second degree term that it may or may not have a term in x to the first power that it may or may not have a constant term. The general form of a quadratic equation is therefore $ax^2+bx+c=0$ where $a\neq0$, with emphasis on the hypothesis $a\neq0$, but with no hypothesis on b and c. The different types of quadratic equations should be amply illustrated, such examples as $x^2=0$, $2x^2-3=0$, $x^2-7x=0$, $x^2-7x+4=0$ being cited.

The student should be taught to solve the quadratic equation by *one* method, and that the method of putting all the terms on the right hand side of the equality mark and factoring the left hand member. As a matter of fact all the methods amount to this fundamentally.

Begin with the simplest case, namely

Case I. $b=0$, $c=0$. The quadratic has the form $ax^2=0$, that is, $a\cdot x\cdot x=0$. Since $a\neq0$, in view of the principle emphasized above, either the first x, if you please to so designate it, equals zero, or the second x equals zero. The essential thing is to accept zero twice as a root of the equation. Note in passing that a root occurring twice as in this case is called a *double* root. The generalization to the case of 0 being a triple root of $x^3=0$, "a" being a quadruple root of $(x-a)^4=0$ interests the student as well as gives him information he has not had before. In fact he might be told, without proof, that an equation has as many roots as its degree. This case under consideration is described as the case in which both roots equal zero.

Case II. $b\neq 0$, $c=0$. The equation then has the form $ax^2+bx=0$, and is therefore equivalent to $x(ax+b)=0$. The fundamental principle cited above gives therefore $x=0$, or $x=-b/a$. This case is described as the case of one root equal to zero, and one not equal to zero.

Case III. $b=0$, $c\neq 0$. The quadratic has the form $ax^2+c=0$, which may be written $a(x^2-c/a)=0$, and factoring it assumes the form $a(x-\sqrt{c/a})\ (x+\sqrt{c/a})=0$. The fundamental principle relative to a product being equal to zero, gives $x=\sqrt{c/a}$, $x=-\sqrt{c/a}$. This case is described as the case of the roots being numerically equal but opposite in sign.

Case IV. $b\neq 0$, $c\neq 0$. This is the general case, the quadratic having the form $ax^2+bx+c=0$. State again by way of emphasis that the very same method, namely the factorization of the left-hand number, can be used. There are two cases to be considered.

1.° The case in which the trinomial expression ax^2+bx+c can be factored by inspection. For example $x^2-7x+6=0$ is equivalent to $(x-6)\ (x-1)=0$, and hence the roots of the equation are 6 and 1. Again, $2x^2-5x-3=0$ is equivalent to $(2x+1)\ (x-3)=0$, and has therefore the roots $-\frac{1}{2}$ and 3. Many problems in which the left hand member can be factored by inspection should be given. That the student should spend a fair amount of time trying to factor by inspection the trinomial involved in a given problem should be insisted upon, for in the long run much time will be saved, to say nothing of that being the proper procedure.

2.° We have now come to the case that is generally presented to the student much too soon, namely the case known as the solution by completing the square. If the text-book does not present this as a mere application of a case in factoring, it should be so presented. That is, it is well to preface this case by considerable drill in factoring trinomial expressions that cannot be factored by inspection. The general case is ax^2+bx+c, which is identically $a(x^2+\frac{b}{a}x+\frac{c}{a})$, which is identically

$$a\left\{(x^2+\frac{b}{a}x+\frac{b^2}{4a^2})-(\frac{b^2}{4a^2}-\frac{c}{a})\right\},$$ which in turn is identically

$$a\left\{(x+\frac{b}{2a})^2-(\frac{\sqrt{b^2-4ac}}{2a})^2\right\}.$$ Thus any trinomial of this type can be written as the difference of two squares, and therefore, in this case, it is equivalent to $a\left\{x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\right\}$ $\left\{x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}\right\}$. To be sure the factors are in general not rational, but the process is none the less correct. Thus the problem of solving $ax^2+bx+c=0$ is the problem of solving

$$a\left\{x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\right\}\left\{x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}\right\}=0.$$ By the same argument used in the simpler cases, the roots are $x=-\frac{b-\sqrt{b^2-4ac}}{2a}$, $x=-\frac{b-\sqrt{b^2-4ac}}{2a}$. Many numerical problems should be solved in order to clarify the process.

It is thought that the presentation of the quadratic equation in this manner involves a minimum amount of machinery, there being only two things to hold in mind, first, how to factor a trinomial (the factorization of a binomial being too simple to mention), second, the fundamental principle *if the product of a number of factors is zero, at least one of the factors is zero.*

In the opinion of the writer the following deductions are within the grasp of the high school student:

First. The nature of the roots of the quadratic. This is important, and strange to say the college freshman seems never to have heard of such a thing. Even in college he hears too often that the discriminant determines the nature of the roots, and hears no hypothesis made on a, b, c when, as a matter of fact, that hypothesis is to be made first. After an attempt to clarify the students' notions of real numbers, that is, rational and irrational numbers, and unreal numbers, the discussion should follow in some such form as:

1.° If a, b, c are rational and

(1) $b^2-4ac>0$, the roots are real and unequal; rational if b^2-4ac is a perfect square, irrational if b^2-4ac is not a perfect square.

(2) $b^2-4ac=0$, the roots are rational and equal.

(3) $b^2-4ac<0$, the roots are unreal and unequal.

2.° If a, b, c are real and

(1) $b^2-4ac>0$, the roots are real and unequal.

(2) $b^2-4ac=0$, the roots are real and equal.

(3) $b^2-4ac<0$, the roots are unreal and unequal.

Nearly all the problems the student has are of the kind 1°, while the classification given in most text-books is 2°, and the student therefore describes the roots less accurately than he may describe them.

Second, the following theorems, which follow immediately:

1°. Every quadratic equation has two roots (proved already by exhibiting them, it being understood that equal roots are counted twice).

2.° If r is a root of the quadratic equation $ax^2+bx+c=0$, $x-r$ is a factor of ax^2+bx+c (proved by noting that the hypothesis means $ar^2+br+c\equiv0$).

3.° A quadratic cannot have a root different from the two exhibited in the solution (proved by supposing it has a root different from these two and showing this leads to a contradiction), and hence a quadratic has two and only two roots.

Austin, January 15, 1917.

THE NUMBER SYSTEM

C. D. RICE

In discussing the solution of the general quadratic, students are asked to use the terms rational, irrational, imaginary, real. These terms have probably been defined in previous lessons, but as a rule the texts on Algebra do not at any one place collect the terms and show their proper relation to each other when applied to numbers. Students without a well defined scheme of the relationship of the meaning of these terms can make no intelligent use of the words in discussions, for instance, the nature of the roots of the quadratic. In order to show the relationship of the different kinds of number the writer has often used the following scheme:

$$
\text{Number} \begin{cases} \text{Real} \begin{cases} \text{Rational} \begin{cases} \text{Integers} \\ \text{Fractions} \end{cases} \\ \text{Irrational} \end{cases} \\ \\ \text{Imaginary} \begin{cases} \text{Pure} \\ \text{Complex} \end{cases} \end{cases}
$$

Before reaching the solution of the quadratic, the pupil should become familiar with this number scheme. It offers a fine opportunity for the teacher to show the class how the number system has grown as the student has advanced in his mathematical studies from primary arithmetic through algebra. Beginning with integers in the primary grades, the necessity soon arose of extending the number system to include fractions. Later certain demands made it necessary to extend the system to include the negative number, the irrational number or surd and later the imaginary number was added. Let the teacher explain the necessity for each expansion of the number notion. The history, traced in this way, in the development of the number notion as shown in elementary arithmetic and algebra may be made very attractive in high school classes. At this point in the development of the number notion, it would be well to have the students define and explain the terms commensurable and incommensurable as applied to numbers and learn their relation to surds and irrational numbers. In a later bulletin, the nature of the negative number will be discussed showing the relation of the negative to the positive number.

SOME OBSERVATIONS ON SOLID GEOMETRY

MARY E. DECHERD

Perhaps no branch of mathematics taught in the high school and college is regarded with as much disfavor by both pupil and teacher as is Solid Geometry. I recently asked more than 150 students who had studied Plane and Solid Geometry, Algebra and Trigonometry in the high school to state which of the subjects they liked best. Only 13, or less than 9 per cent, expressed a preference for Solid Geometry. Moreover of those who arranged the subjects in the order of their preference, by far the majority accorded Solid Geometry a third or fourth place. As these lists were unsigned, there is every reason to believe that the statements made were entirely unprejudiced. Although my statistics have not been accurately compiled, I find that Solid Geometry enjoys a similar lack of favor among instructors.

Because my experience leads me to believe that the above situation exists in many places, I wish to ask the following questions:

1. Why does this condition obtain?
2. Does Solid Geometry deserve such disfavor?
3. What can be done to remedy the situation?

The unpopularity of Solid Geometry is, I think, due mainly to two causes. The first of these is the most serious and the most frequently encountered—lack of familiarity with Plane Geometry. Woe to the teacher and the pupil who inherit such a difficulty! Bad habits of study are often carried over; an inadequate conception of the meaning of "proof," inaccurate notions of the terms used, a failure to learn accurately the statements of Plane Geometry theorems—all these militate against in fact, make impossible—a mastery of Solid Geometry. I once knew a young high school teacher who turned her high senior Solid Geometry class back to Book I in Plane Geometry and the outcome showed that the proceeding, if radical, was wise. The second reason that I would assign is the fact that Solid Geometry requires a certain amount of imagination on the part of the student—an ability to interpret stereoscopic projections. The blind man, strange to say, has the advantage of us here. I have taught several and I find that they can use their "mind's eye" better than those of us who depend more on eyesight. Indeed, space intuition is, for the most part, poorly developed in the average student. This fact occasions at once the difficulty and the opportunity of our subject.

The second question may be put thus: Is Solid Geometry really worth while? Is its content of value? Are its methods specially helpful in educating ("leading out") our students? To all three of these queries I unhesitatingly answer in the affirmative for the folowing reasons:

The mensuration theorems in Books VII and VIII (and IX) are of great practical value. Though these volumes and area theorems are generally not proved rigorously in our high school course, the application of the formulas and the calculation involved give the student much needed practice in ordinary arithmetic processes.

Again, as no where else in elementary mathematics, an opportunity is here afforded of developing space intuition. At first the student must aid his imagination by the use of models. A few slender sticks, a needle and thread, and some card-board must be used, perhaps, to enable the beginner to check the correctness of his interpretation of the configurations with which he is dealing. But soon he should be able to imagine, or mentally picture, the necessary lines and surfaces by means of the drawing without the aid of models.

However, in addition to these two reasons why Solid Geometry is useful in our system of education, a third should be added. This is the stronghold of Geometry and has been from the time of Pythagoras, Plato, and Euclid until this present day. It is, of course, the possible development of the reasoning powers by the use of the logical processes employed in geometric proofs. It was this phase of the subject, I think, that Plato had in mind when he said that "God eternally geometrizes."

The young American, as well as one of more advanced age, is rendered a distinct service when he is made to realize that his own conclusions, even though carefully and honestly thought out and arrived at, are not sufficient justification for their being accepted as convincing. He must consider all the facts in the case; he must assign a satisfactory reason for each step. Moreover, and on the other hand, much breadth of view point, may be developed in the student who is shown beyond the remotest possibility of contradiction that a statement which seemed to him unbelievable is in reality true. What is there beside a mathematical proof that so readily accomplishes these two ends?

While no new type of proof is introduced in Solid Geometry, the complexity of the theorems makes the application of the various types of special value. Three types I should stress. The indirect proof plays a prominent part especially in the uniqueness theorems. I find that few students at first appreciate the real difficulties in the theorem, "One and only one plane can be drawn $\perp$ to a given line through a given point." The instructor can be of service here by emphasizing the fact that such a theorem is worthy of much consideration and effort. Secondly, the locus theorem again comes up for discussion, and should be given the most careful attention of both student and teacher. The student should be required to state the direct, the converse, the opposite, and the opposite-of-the-converse theorems. He should be trained to see that the first and the last, the second and the third have the same content. As the proof given for the following locus theorem is presented so poorly in Wentworth, I append a proof here. (For fig. see Wentworth, p. 301.)

The locus of a point equidistant from the faces of a dihedral angle is the plane bisecting the angle.

I Direct. Every point in the bisecting plane is equidistant from the faces of the angle.

Given plane A M bisecting the dihedral angle D—A O—C.
To prove: Any pt. P in AM equidistant from AD and AC.

Proof. Draw PE and PF (§436, §438).

The plane PEF determines OE OP, and OF (§420).

1. Angle EOP and angle FOP are the plane angles of the dihedrals P—OA—E and P—OA—F respectively (§477, §478, §468).

$\therefore$ Angle EOP=Angle FOP

(If two dihedral $\angle$s are=, their plane $\angle$s are equal.)

2. $\triangle EOP = \triangle FOP$
(rt $\triangle$ with hypothenuse and homologous acute angles=)
$\therefore PE = PF$ (§67).

Q. E. D.

Converse. Every pt. equidistant from the faces of a dihedral angle lies in the plane bisecting the angle.

Given PE=PF, PE and PF being the perpendiculars from P to CA and AD respectively.

To prove PAO is the plane bisecting dihedral angle D—AO—C.

Proof. 1. As in the direct complete the figure and prove angle POF and angle POE the plane angles of P—AO—F and P—AO—E respectively.

2. $\triangle PEO = \triangle PFO$ §89.
$\therefore$ Angle POE=angle POF §67.
$\therefore$ dihedral P—AO—E=dihedral P—AO—F §471.
$\therefore$ PAO is the bisecting plane.

Q. E. D.

$\therefore$ Since every pt. in the bisecting plane is equidistant from the faces of the dihedral angle; and secondly, since every pt. equidistant from the faces of the dihedral angle is in the bisecting plane, the bisecting plane has been proved to be the locus of pts. equidistant from the faces of the dihedral angle.

The third type of proof which should be stressed in Solid Geometry is the limit proof. While several theorems are proved in both Plane Geometry and Solid Geometry by means of limits, I think undoubtedly the most convincing proof is for the following theorem: The volume of a triangular pyramid is the limit of the sum of the volumes of a series of inscribed or circumscribed prism of equal altitudes, when their number is indefinitely increased. This limit should be established prior to the proof of the theorem: "Two triangular pyramids having equivalent altitudes are equivalent"; for the truth of this later theorem is an immediate consequence of the former. The two should not be proved simultaneously, as many geometries do. The fact that an expression for the difference between the variable and its limit can be obtained for the above theorem is the reason why this theorem is specially helpful. I append a proof. Call the pyramid P; the sum of the circumscribed prisms C; the sum of the inscribed prisms I.

Then $P < C$ and $P > I$ or $C > P > I$.

However, $C - I =$ lowest circumscribed prism $=$ the product of its base (B) by its altitude (h). But as h can be diminished at will and as B is constant,

B h can be made as small as we please, or $= e$.

Obviously $C - P < C - I = Bh = e$

$\therefore C - P < e$

$\therefore$ by definition $C = P$ as its limit.

Likewise $I = P$ as its limit.

Q. E. D.

An excellent opportunity to study the interrelations and interdependence of the theorems in Geometry is presented by Book VII. The structure of this book as a whole will give the student a more adequate conception of how geometries are put together.

Again, Solid Geometry contains many theorems which are natural and obvious extensions of Plane Geometry theorems. To illustrate, the theorem concerning the mass center of a tetrahedron bears a close relation to that concerning the centroid, with the concurrence of lines divided in a fixed ratio. Such theorems both impress the Plane Geometry theorems and lead to generalization.

Desargues' famous theorem can easily be proved for space: If the joins of homologons vertices of a $\triangle$ are concurrent, the intersection points of homologous sides are collinear.

In conclusion, I want to suggest a few simple devices which I have found helpful in teaching Solid Geometry.

I asked some well prepared students recently to write out for me to what they attributed their excellence of preparation. I was surprised at their discernment when a number of them frankly stated that frequent written lessons had, more than anything else, held them to their tasks. The grading of these written lessons also plays an important part in raising the standard of work. A theorem that is wrong is not "almost right." Students must be led to see that a chain is no stronger than its weakest link whether it be a chain of iron or a chain of reasoning. An inconclusive proof should be graded zero to enable the student to realize its inadequacy.

Much board work is a strong incentive to the student to be prepared daily. Models should be required from the students for the first theorems in Book VI, and perhaps for some later theorems. The schools should own a few good models. No doubt a good model for the theorem concerning the volume of any parallelopiped would enable it to rid itself of its nickname "the devil's coffin." Figures should be carefully drawn for all theorems. Students should be required to memorize statements of all theorems. Ask the student to see what destruction would be wrought should certain theorems be withdrawn from the book.

The recent experience of a friend of mine is worth considering. A mother realized that her little daughter of eight was doing poor work in arithmetic, though all other work was excellent. Hence she asked the teacher how she might help remove the child's deficiency. Imagine her disappointment when the teacher informed her: "Oh! that is all right. We don't expect little girls to learn Arithmetic. They rarely like it." Are we ever guilty of setting such standards for our students?

"The taste for exactness, the impossibility of contenting one's self with vague notions or of leaning upon mere hypotheses, the necessity for perceiving clearly the connection between certain propositions and the object in view,—these are the most precious fruits of the study of Mathematics."—Lacroix. And all of these fruits should result from the study of Solid Geometry.

WHAT GREAT MEN SAY ABOUT MATHEMATICS

The world of ideas which it (Mathematics) discloses or illuminates, the contemplation of divine beauty and order which it induces, the harmonious connection of its parts, the infinite hierarchy and absolute evidence of the truths with which mathematical science is concerned, these, and such like, are the surest grounds of its title of human regard, and would remain unimpaired were the plan of the universe unrolled like a map at our feet, and the mind of man qualified to take in the whole scheme of creation at a glance.

Sylvester, J. J.

Mathematics . . . the ideal and reason of all careful thinking. Hall, G. Stanley.

Mathematics is the only true Metaphysics.

Thomson, W. (Lord Kelvin).

He who knows not mathematics and the results of recent scientific investigation dies without knowing *truth.*

Schellbach, C. H.

Mathematics, once fairly established on the the foundation of a few axioms and definitons, as upon a rock, has grown from age to age, so as to become the most solid fabric that human reason can boast. Ried, Thomas.

A SWAN SONG

The present number of the Texas Mathematics Teachers' Bulletin closes the second volume. Two years ago the present editors were directed by the Division of Pure and Applied Mathematics to start the infant toddling out among the teachers of Mathematics in the State.

The publication has at least one claim to some distinction. It is the first periodical devoted to mathematics published in Texas. To date, so far as the writer knows, it is the only one.

During these two years the editors have had only one aim always before them—to make the Bulletin intelligible and useful to the teacher of secondary Mathematics. There is no dearth of mathematical publications carrying problems of all sorts and their solution. There are plenty of journals carrying articles that about ten people on earth read and understand. There is a surplus of periodicals devoted to mathematics that contain nothing that the average secondary teacher can make heads or tails of except the advertisements and the information that Dr. So and So has been appoined an instructor in the University of Such and Such. It did not seem to the editors that any addition to the number of unread and unreadable publications was desirable. The files of the Bulletin will show whether this ideal has been adhered to.

For the coming year, Professor A. A. Benett of the School of Pure Mathematics and Mr. H. J. Ettlinger of the School of Applied Mathematics have been selected to guide the destinies of the Bulletin. They are able, enthusiastic, scholarly, and have the teaching of Mathematics very much at heart. We bespeak for them the same friendship that the present editors have enjoyed.

To all the teachers of the state who have helped us by contributions, suggestions, criticisms or even—flattery, we express our sincere thanks. We have done the best that in us lay and according to our lights to help forward the cause of intelligent and effective teaching, we shall continue as we have opportunity to labor in behalf of this same purpose.

If we have preached, it was because we thought we had something to say; if we have found fault, it has been that we might try to point out the remedy; if the "Straight Edge" has been somewhat caustic, it was for the purpose of compelling your attention.

Au Revoir, pax vobiscum, ta ta.

THE EDITORS.

THE STRAIGHT EDGE

Some are born lazy, some get fat, and—some teach geometry.

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Mathematics Teachers' Week, June 13-16.

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Do not let inaccuracy "like a worm i' the bud" devour the damask cheek of your effectiveness in teaching.

* * * * * * * * *

You will be sorry if you miss the Mathematics Teachers' Week, June 13-16.

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"Mournful Numbers" is a phrase originating in the "sad" treatment accorded numbers by most teachers and students of arithmetic.

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You are expected at the Mathematics Teachers' Week, June 13-16.

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In ye olden tyme a student colde sometymes bee fonde that knew wythe some exactnesse one or two definitons. Ye olde order changeth.

* * * * * * * * *

You may get over not coming to the Mathematics Teachers' Week, but you will never look the same. The date is June 13-16.

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"The advance and the perfecting of Mathematics are closely joined to the prosperity of Nations."—Napoleon.

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MATHEMATICS CLUBS

The Editors of the Bulletin desire to impress on all high school teachers of Mathematics the desirability of organizing Mathematical Clubs among the more gifted students under their instruction. Such clubs are a source of interest, and recreation for the better students, arouse a spirit of emulation and make teaching easier and more effective. During the past year Prof. A. A. Bennett organized the Pentegram, an undergraduate club at the University of Texas, with bi-weekly meetings at which were discussed papers and problems both by the students and the staff of instruction. The heaviest duty fell of course on Prof. Bennett, who suggested most of the problems as well as the subjects of most of the papers. His unwearing enthusiasm was contagious and the interest never flagged. The club counted over thirty members and at the close of the term a banquet was held which, though well attended, was not so well attended as the meetings for papers. Any large high school could have such a club and those who count on organizing one can get any needed information by writing to Prof. A. A. Bennett for particulars. In the next number of the Bulletin, Prof. Bennett will outline the character of work which could be profitably undertaken by such high school clubs and give various hints as to the best way of conducting them.